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## **Axiomatization of a preference for most probable winner**

Blavatskyy, Pavlo R

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## AXIOMATIZATION OF A PREFERENCE FOR MOST PROBABLE WINNER

**ABSTRACT.** In binary choice between discrete outcome lotteries, an individual may prefer lottery  $L_1$  to lottery  $L_2$  when the probability that  $L_1$  delivers a better outcome than  $L_2$  is higher than the probability that  $L_2$  delivers a better outcome than  $L_1$ . Such a preference can be rationalized by three standard axioms (solvability, convexity and symmetry) and one less standard axiom (a fanning-in). A preference for the most probable winner can be represented by a skew-symmetric bilinear utility function. Such a utility function has the structure of a regret theory when lottery outcomes are perceived as ordinal and the assumption of regret aversion is replaced with a preference for a win. The empirical evidence supporting the proposed system of axioms is discussed.

**KEY WORDS:** expected utility theory, axiomatization, betweenness, fanning-in, skew-symmetric bilinear utility, regret theory

**JEL CLASSIFICATION:** C91, D81

### 1. INTRODUCTION

An individual has menu-dependent preferences when his preference between two choice options depends on the availability of additional options (content of the choice set). The literature often describes such preferences as “context-dependent” (e.g. Stewart et al. (2003), Tversky and Simonson (1993)). The context of a choice situation is a very general concept, however, that can be also used to describe aspects other than the content of a choice set. A more suitable term to describe a very specific phenomenon—the dependence of individual preferences on the menu of a choice set—is menu-dependence.

In choice under risk (Knight (1921)), a special type of menu-dependent preference is a preference for a lottery that is most probable to outperform all other feasible lotteries. The literature refers to such a preference as “a preference for probabilistically prevailing lottery” (e.g. Bar-Hillel and Margalit (1988)) or “the criterion of the maximum likelihood to be the greatest” (e.g. Blyth, (1972)). Recent experimental evidence suggests that a preference for a most probable winner prevails in binary choice between lottery frequencies of equal expected value (Blavatskyy, 2003) and in small feedback-based problems (e.g. Barron and Erev (2003)), Blavatskyy (2003a). In this paper, I build a system of axioms rationalizing a preference for a most probable winner in binary choice.

Given the probability distributions of any two independent lotteries, it is always possible to calculate directly the (relative) probability of each lottery to outperform the other. Such calculation requires little cognitive effort when the state space (a joint distribution of lotteries) is available. Blavatskyy (2003) provides experimental evidence that a preference for most probable winner emerges when an individual follows a simple majority rule—to pick up a lottery that gives a better outcome in the majority of (equally probable) states of the world. In such a cognitively undemanding environment it is plausible to assume that an individual follows a simple behavioral rule—he calculates the relative probabilities of each lottery to win over the other lotteries and then maximizes among those probabilities. This behavioral rule (the heuristic of relative probability comparisons) resembles one-reason fast and frugal decision making (e.g. Gigerenzer and Goldstein (1996)). Like all heuristics, it ignores some of the available information by treating lottery outcomes as ordinal. Additionally, like all heuristics, this behavioral rule applies only to a bounded subset of decision problems, e.g. when lotteries have equal or similar expected values.

In cognitively demanding environments, a straightforward calculation of relative probabilities of a lottery to win over others, however, demands more cognitive effort. Examples include situations when probability information is presented

visually (e.g. Tversky (1969)) or not presented at all (e.g. Barron and Erev (2003)), when lotteries have many outcomes or an individual faces a choice among many lotteries. Nevertheless, assuming an individual preference for most probable winner it is possible to explain observed decision making in such environments, as demonstrated in Blavatskyy (2003a) in his alternative explanation of the data in Barron and Erev (2003).

Since individuals are likely to use only simple rules of thumb (e.g. Gigerenzer et al. (1999)), a descriptive fit of a preference for most probable winner in cognitively demanding decision environments can be explained only through a general theory of preference. Unlike a heuristic approach that describes a plausible psychological process underlying observed decision making (e.g. Newell and Shanks (2003)), a theory of preference states that an individual has an underlying preference for most probable winner. The purpose of this paper is to explore the theoretical properties of an individual's preference for most probable winner, and how it is related to various non-expected utility theories (e.g. Starmer (2000)). Specifically, the paper explores what normative axioms are necessary and sufficient for rationalizing such preference, and how those axioms accord with the experimental evidence.

The proposed axiomatization provides theoretical insights into an individual's preference for most probable winner and highlights some surprising connections to other decision theories. It also provides "thought experiment" evidence for a descriptive validity of the theory (e.g. Friedman and Savage (1952)), Machina (1982). However, "thought experiment" evidence can be drastically different from actual decision making (e.g. Tversky (1969)). Therefore, the paper also focuses on the experimental evidence supposedly documenting the systematic violation of the proposed axioms.

The remainder of this paper is structured as follows. Section 2 introduces the system of axioms and derives a utility function representation and family of indifference curves. The descriptive validity of the proposed axioms is discussed in section 3. Section 4 concludes.

## 2. THE SYSTEM OF AXIOMS

2.1. *Basic definitions*

An option  $A$  is strictly preferred to option  $B$ , or  $A \succ B$ , if an individual chooses  $A$  and is not willing to choose  $B$  from the choice set  $\{A, B\}$ . An individual is indifferent between choice options  $A$  and  $B$ , or  $A \sim B$ , if the choice of  $A$  and the choice of  $B$  are equally possible from the choice set  $\{A, B\}$ .

This paper deals with individuals' binary choices between discrete lotteries. The set of lottery outcomes  $X = \{x_1, \dots, x_n\}$  is finite and ordered in such a way that  $x_1 < \dots < x_n$ . Outcomes are not necessarily monetary (measured in reals). They are only required to be strictly ordered in terms of subjective preference. A lottery  $L(p_1, \dots, p_n)$  is defined as a mapping  $L: X \mapsto [0, 1]^n$ , where  $p_i \in [0, 1]$  is the probability of occurrence of outcome  $x_i$ ,  $i \in \{1, \dots, n\}$  and  $\sum_{i=1}^n p_i = 1$ .

In a joint independent distribution of any two lotteries  $L_1(p_1, \dots, p_n)$  and  $L_2(q_1, \dots, q_n)$  only three events are possible:  $L_1$  delivers a better outcome than  $L_2$  (state  $s_1$ ),  $L_2$  delivers a better outcome than  $L_1$  (state  $s_2$ ) and lotteries  $L_1, L_2$  deliver the same outcome (state  $s_3$ ). An individual has a preference for most probable winner when Equation (1) holds for any two lotteries  $L_1$  and  $L_2$ . This decision rule is rationalized below.

$$L_1 \succ L_2 \Leftrightarrow \text{prob}(s_1) > \text{prob}(s_2) \Leftrightarrow \sum_{i=1}^{n-1} q_i \left( 1 - \sum_{j=1}^i p_j \right) > \sum_{i=1}^{n-1} p_i \left( 1 - \sum_{j=1}^i q_j \right) \quad (1)$$

In the remainder of this paper each pair of lotteries is assumed to be statistically independent. This assumption is not restricting the normative or descriptive applications of the model. A preference for most probable winner is easily extendable on the domain of acts (Savage (1954)) In a binary choice between two acts an individual recodes the outcome

of each act as a relative gain (e.g. +1) or a relative loss (e.g. -1) and then chooses the act that yields a relative gain with the highest probability. Such rule of thumb is intuitively plausible and cognitively undemanding. Therefore, there is no apparent reason for axiomatizing such preference when a joint distribution of lotteries (state space) is given. On the contrary, a preference for most probable winner over independent lotteries is not immediately appealing. This is the main reason why this axiomatization is restricted to independent lotteries.

## 2.2. *Standard axioms*

This section presents a set of axioms that were already used in other axiomatizations of decision theories (e.g. Fishburn (1982), (1988)). Notably, this set of standard axioms does not contain the transitivity axiom. Indeed, a preference for most probable winner can be intransitive (e.g. Blyth (1972)). Intuitively, the structure of such preference is foremost based on the relative (binary) comparisons rather than on a separate (menu-independent) evaluation of lotteries. For example, lottery  $L_1$  that yields €4 with probability  $\Psi = (\sqrt{5} - 1)/2 \approx 0.618$  and €1 otherwise is more probable to deliver a higher outcome than €3. Lottery  $L_2$  that yields €2 with probability  $\Psi$  and €5 otherwise is more probable to deliver a lower outcome than €3. However,  $L_2$  delivers a higher outcome than  $L_1$  with probability  $\Psi$ . Hence, a preference for most probable winner does not necessarily impose a transitive order on lotteries.

*Axiom 1 (Solvability).* For any three lotteries  $L_1, L_2, L_3$  such that  $L_1 \succ L_2$  and  $L_2 \succ L_3$  there is a number  $\alpha \in (0, 1)$  such that  $\alpha L_1 + (1 - \alpha) L_3 \sim L_2$

*Axiom 2 (Convexity).* For any three lotteries  $L_1, L_2, L_3$  and for any number  $\alpha \in (0, 1)$ :

- (a) if  $L_1 \succ L_2$  and  $L_1 \geq L_3$  then  $L_1 \succ \alpha L_2 + (1 - \alpha) L_3$ ,
- (b) if  $L_1 \sim L_2$  and  $L_1 \sim L_3$  then  $L_1 \sim \alpha L_2 + (1 - \alpha) L_3$ ,
- (c) if  $L_1 \succ L_2$  and  $L_3 \geq L_2$  then  $\alpha L_1 + (1 - \alpha) L_3 \succ L_2$ .

*Axiom 3 (Symmetry).* For any three lotteries  $L_1, L_2, L_3$  such that  $L_1 \succ L_2$ ,  $L_2 \succ L_3$ ,  $L_1 \succ L_3$ ,  $L_2 \sim 0.5L_1 + 0.5L_3$  and for any number  $\alpha \in (0, 1)$ :

$\alpha L_1 + (1 - \alpha) L_3 \sim 0.5L_1 + 0.5L_2$  if and only if  $\alpha L_3 + (1 - \alpha) L_1 \sim 0.5L_3 + 0.5L_2$ .

Fishburn (1982, 1988) proved the following theorem.

**THEOREM 1.** *Axioms 1–3 hold if and only if there is a skew-symmetric function  $\psi : X \times X \rightarrow R$  (unique up to a multiplication by a positive constant) such that for any two lotteries  $L_1(p_1, \dots, p_n)$  and  $L_2(q_1, \dots, q_n)$ :  $L_1 \succ L_2$  if and only if*

$$\sum_{i=1}^n \sum_{j=1}^n p_i q_j \psi(x_i, x_j) > 0.$$

### 2.3. A fanning-in axiom

A fanning-in axiom assumes a particular type of diminishing sensitivity to probability. Specifically, when probability mass is largely shifted to the best or the worst outcome, tiny probabilities attached to the intermediate outcomes become progressively unimportant for decision. This axiom has not been used in other axiomatizations in the literature.

*Axiom 4 (A fanning-in).* For any lottery  $L_1$  that delivers  $x_i$  with probability  $p_i$  and  $x_n$  otherwise, and for any lottery  $L_2$  that delivers  $x_j \succ x_i$  with probability  $p_j$  and  $x_n$  otherwise, such that  $L_1 \sim L_2$ :  $\lim_{p_i \rightarrow 0} p_j / p_i = 1$ . For any lottery  $L_3$  that delivers  $x_k$  with probability  $p_k$  and  $x_1$  otherwise, and for any lottery  $L_4$  that delivers  $x_l \succ x_k$  with probability  $p_l$  and  $x_1$  otherwise, such that  $L_3 \sim L_4$ :  $\lim_{p_l \rightarrow 0} p_k / p_l = 1$ .

To understand the logic behind axiom 4, consider first the situation when  $p_i \rightarrow 0$ . First of all, notice that  $p_j > p_i$  because  $x_i \prec x_j \prec x_n$ . If  $p_i \rightarrow 0$ , lottery  $L_1$  approaches to the lottery  $\bar{L}(0, \dots, 0, 1)$ , which gives the best possible outcome  $x_n$  for sure. Since there could be no other lottery  $\tilde{L}_2$  such that  $\tilde{L}_2 \sim \bar{L}$  it must be the case that  $\lim_{p_i \rightarrow 0} p_j = 0$ . When two lotteries  $L_1, L_2$  approach to the lottery  $\bar{L}$  the absolute

differences in tiny probabilities attached to the not-best outcome disappear. Axiom 4 additionally requires that the relative differences in probabilities attached to the not-best outcome also disappear as  $L_1$  and  $L_2$  become increasingly similar to  $\bar{L}$  i.e.  $\lim_{p_i \rightarrow 0} p_j / p_i = 1$ .

Expected utility theory violates axiom 4 because it implies that  $\lim_{p_i \rightarrow 0} p_j / p_i > 1$ . When  $\lim_{p_i \rightarrow 0} p_j / p_i = 1$  then an individual's indifference curves plotted in the probability triangle<sup>1</sup> (e.g. Machina, 1982) are not parallel but fanning-in, which explains the name of the axiom. The assumption  $\lim_{p_i \rightarrow 0} p_j / p_i = 1$  also implies that an individual becomes infinitely risk seeking when probability mass is largely shifted to the best outcome. Notice that axiom 4 is stronger than the reversed definition of fanning-out (hypothesis II) proposed by Machina (1982).

The second part of axiom 4 assumes that the above logical argument applies as well to the situation when lotteries  $L_3(1 - p_k, 0, \dots, 0, p_k, 0, \dots, 0)$  and  $L_4(1 - p_l, 0, \dots, 0, p_l, 0, \dots, 0)$ ,  $L_3 \sim L_4$ , approach to the lottery  $\underline{L}(1, 0, \dots, 0)$ , which gives the worst possible outcome  $x_1$  for sure. The only difference is that an individual becomes infinitely risk averse when probability mass is largely shifted to the worst outcome. The implication of axiom 4 that an individual becomes risk seeking (averse) when probability mass is largely shifted to the best (worst) outcome is the counterpart of Machina's intuition for universal fanning out (Machina, 1987, pp. 129–130). As shown in the proof of theorem 2 below the intuitive role of axiom 4 is “to erase” the cardinal difference between lottery outcomes. An individual who maximizes the probability of a relative gain ignores the information about the size of this gain, i.e. he or she treats lottery outcomes in an ordinal way.

**THEOREM 2.** *Axioms 1–4 hold if and only if Equation (1) holds for any two lotteries  $L_1$  and  $L_2$ .*

Proof is presented in the appendix.

Theorem 2 implies that an individual's preference for most probable winner is a special case of the skew-symmetric bilin-



ear utility theory (e.g. Fishburn, 1982, 1988). The addition of a fanning-in axiom restricts a general skew-symmetric bilinear functional derived by Fishburn so that only the ordinal difference in lottery outcomes is taken into account. When lotteries are distributed independently, skew-symmetric bilinear utility theory coincides with regret theory (e.g. Loomes and Sugden, 1982, 1987). When the anticipated net advantage function of regret theory is ordinal in outcomes (e.g. Equation (4) in the appendix), the decision rule of regret theory reduces to a preference for most probable winner. However, such “ordinal” function  $\psi(x_i, x_j)$  always violates a key assumption of regret theory, regret aversion, which requires  $\forall x > y > z \Rightarrow \psi(x, z) > \psi(x, y) + \psi(y, z)$  (e.g. Loomes et al., 1992). In terms of regret theory the “ordinal” function  $\psi(x_i, x_j)$  of a preference for most probable winner always reflects regret seeking:  $\forall x > y > z \Rightarrow \psi(x, z) < \psi(x, y) + \psi(y, z)$ .

Figure 1 plots the map of the indifference curves representing a preference for most probable winner inside the probability triangle (Machina, 1982). The same map of indifference curves is implied by the weighted utility theory when the weight of the medium outcome is greater than unity (e.g. Chew and Waller, 1986). Notice that this indifference map is independent of individual-specific parameters (functions) and cardinal measures of lottery outcomes, i.e. the map is invariant

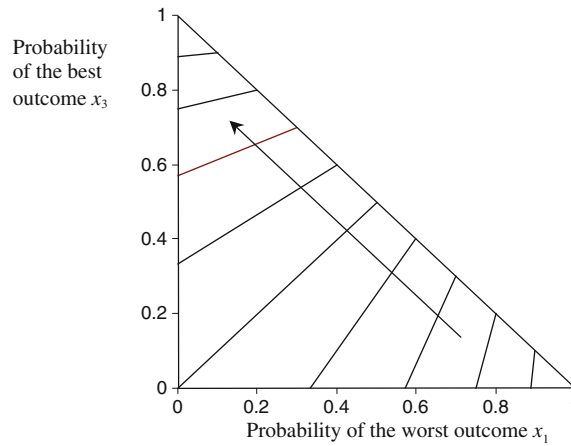


Figure 1. Family of indifference curves inside the probability triangle.

for all triples of lottery outcomes such that  $x_1 < x_2 < x_3$ . The family of indifference curves implied by axioms 1–4 consists of straight lines with different slopes reflecting a changing individual attitude towards risk. Specifically, a universal fanning-in, as in figure 1, shows that an individual becomes more risk seeking (averse) when probability mass is shifted to the best (worst) outcome, which Chew and Waller (1986) call “the heavy hypothesis”. Figure 1 shows that indifference curves do not intersect inside the probability triangle, i.e. preferences are transitive. In a special case when  $n = 3$ , axioms 1–4 imply a transitivity axiom (see working paper Blavatskyy (2004) for a formal proof).

Figure 1 demonstrates that an individual is risk neutral along the 45° line on figure 1, i.e. he or she is exactly indifferent between a medium outcome for sure and a 50%-50% chance of the best and the worst outcome. This is a direct consequence of the symmetry axiom. The symmetry axiom probably gains the most of its intuitive appeal when lottery outcomes are “similar” to each other. Hence, a preference for most probable winner is especially appealing when lotteries have equal or similar expected values.

### 3. DESCRIPTIVE VALIDITY OF PROPOSED AXIOMS

This section critically reviews the existing experimental evidence on the alleged violations of the four proposed axioms. To the best of my knowledge, there are no studies documenting any systematic violations of axiom 1 (solvability).

#### 3.1. *Violations of betweenness*

Part b) of Axiom 2 (convexity) implies a betweenness axiom. According to this axiom if an individual is indifferent between two lotteries, then their probability mixture is equally good. Systematic violations of the betweenness are reported in Coombs and Huang (1976), Chew and Waller (1986), Camerer (1989), Battalio et al. (1990), Gigliotti and Sopher (1993) and

Camerer and Ho (1994). However, Blavatskyy (2005) recently demonstrated that these experimental findings can be alternatively attributed to the effect of random errors.

An asymmetric split between revealed quasi-concave and quasi-convex preferences was conventionally interpreted as evidence of the violations of the betweenness. Blavatskyy (2005) shows that it can be caused by random errors unless the modal choice pattern is inconsistent with the betweenness. The latter case is documented only in few studies. Prelec (1990) finds that 76% of subjects reveal quasi-concave preferences and only 24% of subjects respect betweenness when the probability mass of the hypothetical lotteries is largely shifted to the worst outcome. Camerer and Ho (1994) replicate this result for one lottery triple “TUV” with real payoffs. Bernasconi (1994) finds a strong asymmetric violation of betweenness when betweenness is not a modal choice pattern in two lottery pairs (1 and 3).

### 3.2. *Evidence for fanning-in*

A survey of experiments testing the shape of individuals' indifference curves suggests that there is a non-negligible evidence for fanning-out going back to the Allais paradox (e.g. Allais, 1953) and common consequence and common ratio effects (Starmer, 2000). However, a universal fanning-out hypothesis (Machina, 1982) is rejected. There is a growing evidence that supports a universal fanning-in. This new evidence suggests that indifference curves tend to fan in when the probability mass is associated with the best and the worst outcome and tend to fan out when the probability mass is associated with medium outcomes.<sup>2</sup> In addition, the evidence for fanning-in in all regions of the probability triangle has recently emerged.

Conlisk (1989) finds strong experimental support for the type of fanning-in implied by axiom 4—53% and 80% of subjects choose a more risky gamble in a common consequence problem when probability mass is largely shifted to the medium and the best outcome, correspondingly. This finding can be interpreted as an individual's indifference curves

becoming almost horizontal when probability mass is largely shifted to the best outcome. Analogously, the so-called vertical fanning-in is documented in Starmer and Sugden (1989), Camerer (1989) p. 92 and Battalio et al. (1990). Wu and Gonzalez (1998) p. 119 report a vertical fanning-in when the probability of the best outcome is above 0.33 and a vertical fanning-out when it is below 0.33.

Prelec (1990) and Kagel et al. (1990) find fanning-in when probability mass is largely shifted to the worst outcome. Wu and Gonzalez (1996) report the so-called horizontal fanning-in when a probability of the worst outcome is above 0.63 and a horizontal fanning-out when it is below 0.63. Camerer (1989) p. 92 finds a similar evidence for small gains.

Bernasconi (1994) p. 63 finds an experimental evidence for fanning-in by observing a reverse common ratio effect. Cubitt and Sugden (2001), Bosman and van Winden (2001) and Cubitt et al. (2004) find an indirect evidence for a reverse common ratio effect in dynamic decision making under risk. Barron and Erev (2003) find a reverse common ratio effect in small feedback-based decision making. Battalio et al. (1990) and Thaler and Johnson (1990) find an evidence for fanning-in, i.e. an increased risk seeking for stochastically dominant lotteries when lotteries involve only guaranteed gains. Finally, Starmer (1992) and Humphrey and Verschoor (2004) find strong evidence consistent with a universal fanning-in in all regions of the probability triangle.

The above literature elicits fanning-in/out of an individual's indifference curves from an observed binary choice in a common consequence or common ratio problem involving lotteries typically defined on a common three-outcome structure. Therefore, the main findings from this literature can be summarized in the probability triangle presented in Figure 2.

#### 4. CONCLUSIONS

The proposed axiomatization explores theoretical features of an individual's preference for most probable winner in a binary

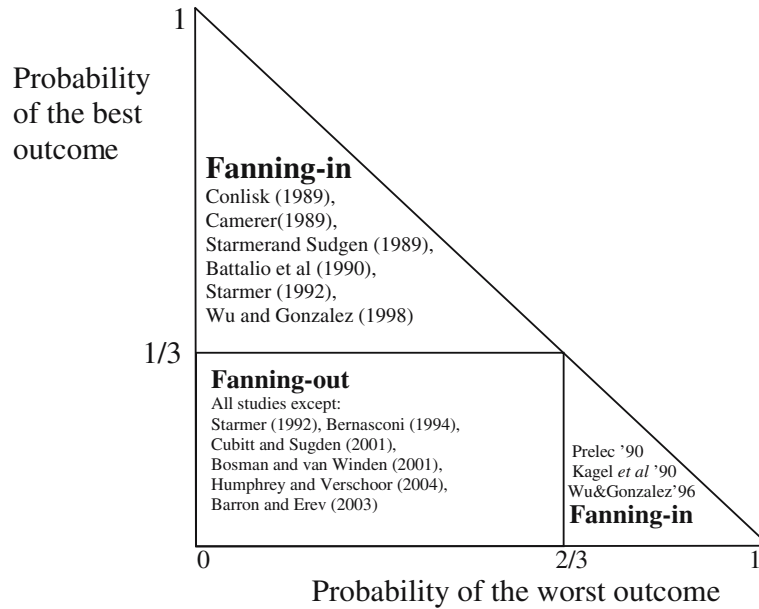


Figure 2. Empirical evidence for fanning-in.

choice under risk. Although such preference is implied by a simplistic behavioral rule (the heuristic of relative probability comparisons), I find some surprising perhaps even unexpected connections with other decision theories (skew-symmetric bilinear utility, weighted utility and regret theory). A preference for most probable winner is rationalized by four axioms: solvability, convexity, symmetry and a fanning-in. Notably, transitivity of preferences is not required. The present paper deals with binary choice; a natural extension of this work is to axiomatize a preference for most probable winner in a choice among many lotteries.

A preference for most probable winner falls into the betweenness class of decision theories that assume the linearity in probability of the sets  $\{L : L \sim L_0\}$ ,  $\forall L_0$ . The alleged systematic violations of betweenness found in the experimental literature in the late 1980's and early 1990's can be explained within the concept of a stochastic utility developed in the mid 1990's. If the experimental evidence is reevaluated in the light of notions

of stochastic utility, the betweenness axiom turns out to be quite descriptive.

Experimental evidence also emerges for universal fanning-in of indifference curves. However, this evidence seems to be stronger for some areas of the probability triangle than for others. The experimental evidence for the system of axioms proposed here to rationalize an individual's preference for most probable winner provides indirect evidence for the domain of applicability of the heuristic of relative probability comparisons.

A preference for most probable is a special case of a skew-symmetric bilinear utility theory and regret theory when outcomes are perceived as ordinal and the assumption of regret aversion is replaced with a preference for a win. Thus, an individual's preference for most probable winner is a simplified mirror image of regret theory.

## APPENDIX

*Proof of Theorem 2.* According to Theorem 1, axioms 1–3 hold if and only if there is a skew-symmetric function  $\psi(.,.)$  such that for any two lotteries  $L_1(p_1, \dots, p_n)$  and  $L_2(q_1, \dots, q_n)$ :  $L_1 \succ L_2 \Leftrightarrow \sum_{i=1}^n \sum_{j=1}^n p_i q_j \psi(x_i, x_j) > 0$ . For a specific pair of lotteries  $L_1$  and  $L_2$  described in the first part of axiom 4 the last statement can be rewritten as Equation (2).

$$L_1 \sim L_2 \Leftrightarrow p_i p_j \psi(x_i, x_j) + p_i (1 - p_j) \psi(x_i, x_n) + (1 - p_i) p_j \psi(x_n, x_j) + (1 - p_i) (1 - p_j) \psi(x_n, x_n) = 0. \quad (2)$$

The right-hand side of (2) can be rewritten as equation (3).

$$p_j = \frac{p_i \psi(x_i, x_n) + (1 - p_i) \psi(x_n, x_n)}{p_i \psi(x_i, x_n) + (1 - p_i) \psi(x_n, x_n) - p_i \psi(x_i, x_j) - (1 - p_i) \psi(x_n, x_j)}. \quad (3)$$

Taking the limit from both sides of (3) when  $p_i \rightarrow 0$  we obtain that  $\lim_{p_i \rightarrow 0} p_j = 0$  only if  $\psi(x_n, x_n) = 0$ . Furthermore,  $\lim_{p_i \rightarrow 0} p_j / p_i = 1$  if and only if  $\psi(x_i, x_n) = -\psi(x_n, x_j) =$

$\psi(x_j, x_n)$  with the latter equality due to the skew-symmetric property of function  $\psi(.,.)$ . Following the same argument for a pair of lotteries  $L_3$  and  $L_4$  described in the second part of axiom 4 we obtain that  $\psi(x_1, x_l) = \psi(x_1, x_k)$ ,  $\forall k, l \in \{2, \dots, n\}$ . Function  $\psi(.,.)$  then has the following form:

$$\psi(x_i, x_j) = \begin{cases} a & i > j, \\ 0 & i = j, \\ -a & i < j, \end{cases} \quad (4)$$

where  $a > 0$  is constant. Intuitively, the addition of axiom 4 imposes ordinality on the Fishburn's function  $\psi(.,.)$ . Thus, axioms 1–4 hold if and only if for any two lotteries  $L_1(p_1, \dots, p_n)$  and

$$L_2(q_1, \dots, q_n) : L_1 \succ L_2 \Leftrightarrow \sum_{i=1}^n \sum_{j=1}^n p_i q_j \psi(x_i, x_j) > 0,$$

where function  $\psi(.,.)$  is defined by equation (4). This last statement is algebraically equivalent to equation (1). *Q.E.D.*

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#### NOTES

1. Note that lotteries  $L_1, L_2$ , although defined as probability distributions over  $n$  outcomes, have non-zero probabilities attached only to three outcomes  $x_i, x_j, x_n$ . Therefore, lotteries  $L_1, L_2$  can be plotted in the probability triangle based only on outcomes  $x_i, x_j, x_n$ .
2. Neilson (1992) develops a theory that accommodates this experimental evidence.

## REFERENCES

- Allais, M. (1953), Le comportement de l'homme rationnel devant le risque: critique des postulats et axiomes de l'école Américaine, *Econometrica* 21, 503–546.
- Bar-Hillel, M. and Margalit, A. (1988), How vicious are cycles of intransitive choice? *Theory and Decision* 24, 119–145.
- Barron, G. and Erev, I. (2003), Small feedback-based decisions and their limited correspondence to description-based decisions, *Journal of Behavioral Decision Making* 16, 215–233.
- Battalio, R., Kagel, J. and Jiranyakul, K. (1990), Testing between alternative models of choice under uncertainty: Some initial results, *Journal of Risk and Uncertainty* 3, 25–50.
- Bernasconi, M. (1994) Nonlinear preference and two-stage lotteries: theories and evidence, *Economic Journal* 104, 54–70.
- Blavatsky, P. (2003), Content-dependent preferences in choice under risk: heuristic of relative probability comparisons, IIASA Interim Report IR-03–031 [www.iiasa.ac.at/Publications/Documents/IR-03-031.pdf](http://www.iiasa.ac.at/Publications/Documents/IR-03-031.pdf)
- Blavatsky, P. (2003a), Note on “Small feedback-based decisions and their limited correspondence to description-based decisions”, CERGE-EI Working Paper#218 [www.cerge-ei.cz/pdf/wp/Wp218.pdf](http://www.cerge-ei.cz/pdf/wp/Wp218.pdf)
- Blavatsky, P. (2004), Axiomatization of a preference for most probable winner, CERGE-EI Working Paper#226 <http://www.cerge-ei.cz/pdf/wp/Wp226.pdf>
- Blavatsky, P. (2005), Violations of betweenness or random errors? *Economics Letters*, forthcoming.
- Blyth, C. (1972), Some probability paradoxes in choice from among random alternatives, *Journal of the American Statistical Association* 67, 366–382.
- Bosman, R. and van Winden, F. (2001), “Anticipated and experienced emotions in an investment experiment”, Tinbergen Institute Discussion Paper No 01–058/1.
- Camerer, C. (1989), An experimental test of several generalized utility theories, *Journal of Risk and Uncertainty* 2, 61–104.
- Camerer, C. (1992), Recent tests of generalizations of expected utility theory, In *Utility: Theories, Measurement, and Applications*. W. Edwards (ed.), Kluwer, 207–251.
- Camerer, C. and Ho, T. (1994), “Violations of the betweenness axiom and nonlinearity in probability”, *Journal of Risk and Uncertainty* 8, 167–196.
- Chew, S. and Waller, W. (1986), Empirical tests of weighted utility theory, *Journal of Mathematical Psychology* 30, 55–72.
- Conlisk, J. (1989), Three variants on the Allais example, *American Economic Review* 79, 392–407.



- Coombs, C. and Huang, L. (1976), Tests of the betweenness property of expected utility, *Journal of Mathematical Psychology* 13, 323–337.
- Cubitt, R. and Sugden, R. (2001), Dynamic decision-making under uncertainty: an experimental investigation of choices between accumulator gambles, *Journal of Risk and Uncertainty* 22, 103–28.
- Cubitt, R., Starmer, Ch. and Sugden, R. (2004), Dynamic decisions under uncertainty: some recent evidence from economics and psychology, in I. Brocas, and J. Carrillo (eds.), *The Psychology of Economic Decisions: Reasons and Choices (Vol. 2)*, OUP, Oxford, pp. 81–110.
- Fishburn, P. (1982), Nontransitive measurable utility, *Journal of Mathematical Psychology* 26, 31–67.
- Fishburn, P. (1988), *Nonlinear Preference and Utility Theory*, Wheatsheaf Books Ltd.
- Friedman, F. and Savage L. (1952), The expected-utility hypothesis and the measurability of utility, *Journal of Political Economy* LX, 463–474.
- Gigerenzer, G. and Goldstein, D. (1996), Reasoning the fast and frugal way. Models of bounded rationality, *Psychological Review* 103, 650–669.
- Gigerenzer, G., Todd, P. and the ABC Research Group (1999), *Simple Heuristics That Make Us Smart*, Oxford University Press, Oxford.
- Gigliotti, G. and Sophor, B. (1993), A test of generalized expected utility theory, *Theory and Decision* 35, 75–106.
- Humphrey, S. and Verschoor, A. (2004), Decision-making under risk among small farmers in East Uganda, *Journal of African Economies*, 13 44–101.
- Kagel, J., MacDonald, D., Battalio, R. (1990), Tests of ‘fanning out’ of indifference curves: results from animal and human experiments, *American Economic Review* 80, 912–921.
- Knight, F. (1921), *Risk, Uncertainty, and Profit*, Houghton Mifflin, New York.
- Loomes, G., Starmer, Ch. and Sugden, R. (1992), Are preferences monotonic? Testing some predictions of regret theory, *Economica* 59, 17–33.
- Loomes, G. and Sugden, R. (1982), Regret theory: an alternative theory of rational choice under uncertainty, *Economic Journal* 92, 805–824.
- Loomes, G. and Sugden, R. (1987), Some applications of a more general form of regret theory, *Journal of Economic Theory* 41, 270–287.
- Loomes, G. and Sugden, R. (1995), Incorporating a stochastic element into decision theories, *European Economic Review* 39, 641–648.
- Loomes, G. and Sugden, R. (1998), Testing different stochastic specifications of risky choice, *Economica* 65, 581–598.
- Machina, M. (1982), ‘Expected utility’ analysis without the independence axiom, *Econometrica* 50, 277–323.
- Machina, M. (1987), ‘Choice under uncertainty: problems solved and unsolved’, *Journal of Economic Perspectives* 1, 121–154.

- Neilson, W. (1992), A mixed fan hypothesis and its implications for behavior toward risk, *Journal of Economic Behavior and Organization* 19, 197–211.
- Newell, B. and Shanks, D.R. (2003), Take the best or look at the rest? Factors influencing “one-reason” decision making, *Journal of Experimental Psychology: Learning, Memory, & Cognition* 29, 53–65.
- Prelec, D. (1990), A ‘pseudo-endowment’ effect, and its implications for some recent nonexpected utility models, *Journal of Risk and Uncertainty* 3, 247–259.
- Savage, L.J. (1954), *The Foundations of Statistics*. John Wiley, New York.
- Starmer, Ch. (1992), Testing new theories of choice under uncertainty using the common consequence effect, *Review of Economic Studies* 59, 813–830.
- Starmer, Ch. (2000), Developments in non-expected utility theory: the hunt for a descriptive theory of choice under risk, *Journal of Economic Literature* 38, 332–382.
- Starmer, Ch. and Sugden, R. (1989), Probability and juxtaposition effects: An experimental investigation of the common ratio effect, *Journal of Risk and Uncertainty* 2, 159–178.
- Thaler, R. and Johnson, E. (1990), Gambling with the house money and trying to break even: the effects of prior outcomes on risky choice, *Management Science* 36, 643–661.
- Tversky, A. (1969), Intransitivity of preference, *Psychological Review* 76, 31–48.
- Tversky, A. and Simonson, I. (1993) Context-dependent preferences, *Management Science* 39, 1179–1189.
- Wu, G. and Gonzalez, R. (1996), Curvature of the probability weighting function, *Management Science* 42, 1676–90.
- Wu, G. and Gonzalez, R. (1998), Common consequence conditions in decision making under risk, *Journal of Risk and Uncertainty* 16, 115–139.

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